

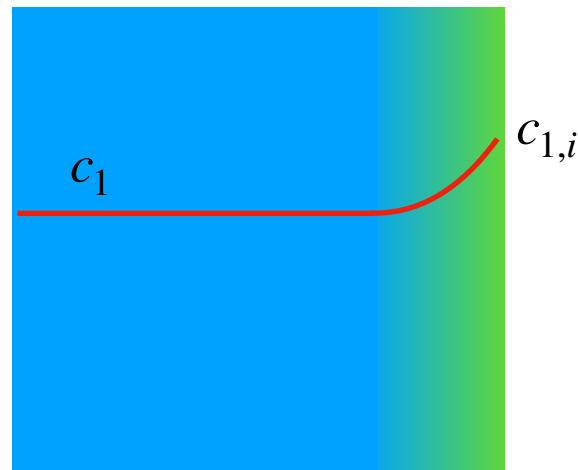
ChE-402: Diffusion and Mass Transfer

Lecture 6

Intended Learning Outcome

- ✦ Compare the concept of diffusion and mass transfer.
- ✦ Analysis of a few mass transfer cases using mass transfer coefficient.
- ✦ Derive overall mass transfer coefficient based on interfacial equilibrium.

The concept of mass transfer coefficient



Mass transfer coefficient is empirical but is convenient

(knowledge of bulk concentrations may be sufficient)

$$\text{Flux} = \text{Mass transfer coefficient} * (\Delta c)$$

$$N_1 = k(c_{1,i} - c_1)$$

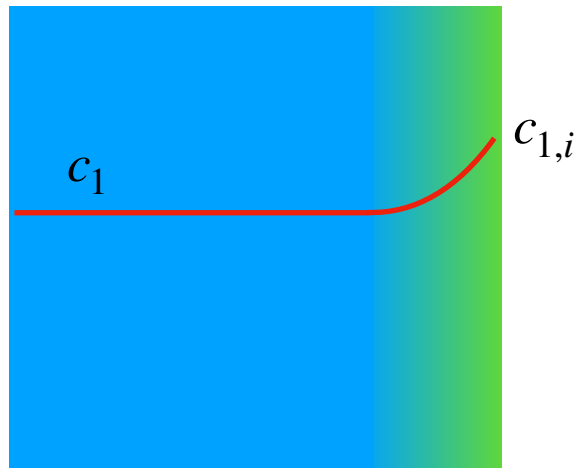
Diffusion coefficient is fundamental but requires knowledge of concentration gradient

(e.g. we need to know precisely the thickness of film)

$$\text{Flux} = \text{Diffusion coefficient} * (-\nabla c)$$

$$J_1 = -D \nabla c_1$$

Compare the unit of k and D



$$N_1 = k(c_{1,i} - c_1)$$

Units of k = m/s

$$J_1 = -D \nabla c_1$$

Units of D = m²/s

- ✦ Mass transfer coefficient is similar to the rate constant of reaction, written per area, not per volume.
- ✦ Because its unit is same as that of velocity, k is sometime referred to as 'velocity of diffusion'.

When should we use mass transfer coefficient and when should we use diffusion coefficient

k

Engineering approximation

Complex geometry

Well mixed bulk concentrations

When we are interested in carrying out quick/approximate estimates of mass transport

D

Fundamental transport property

Geometry not too complex (knowledge of length)

Bulk is not well-mixed (transient slab problem)

When we are interested in understanding relative contribution from diffusion and convection

Mass transfer coefficient can evolve with time

Consider diffusion across a semi-infinite slab

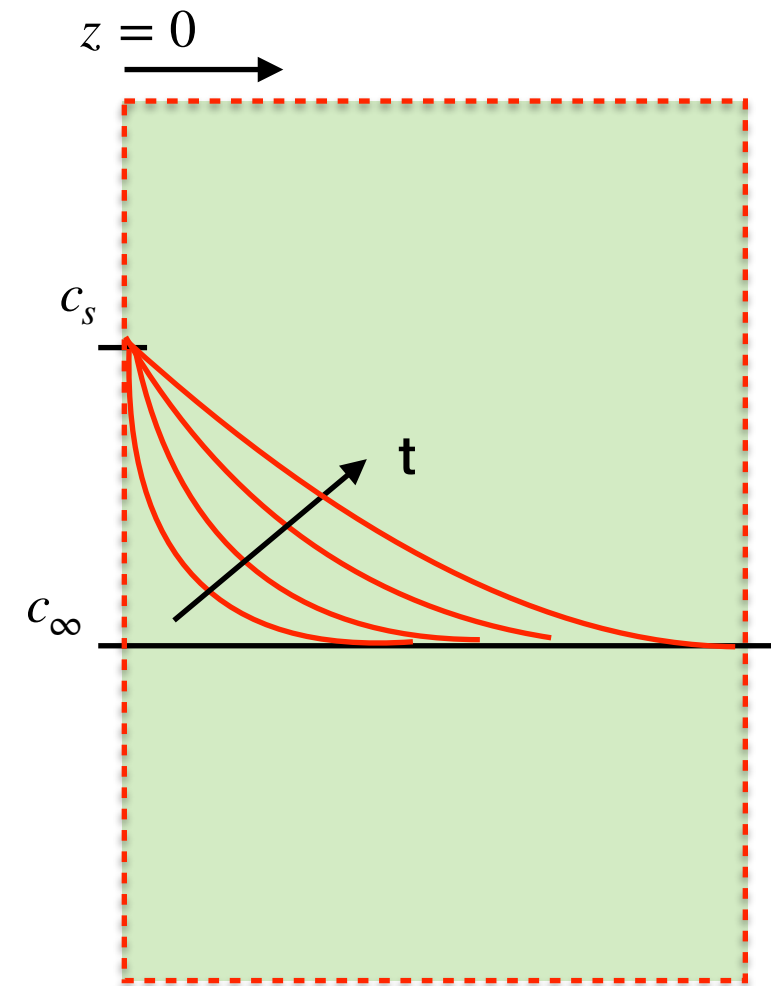
$$J_{surface} = \sqrt{\frac{D}{\pi t}}(c_s - c_\infty) = k(c_s - c_\infty)$$

$$\Rightarrow k = \sqrt{\frac{D}{\pi t}}$$

- ✦ k is initially (t = 0) quite large, but reduces with time.
- ✦ If you have to use k, a time averaged k should be used.

$$\bar{k} = \frac{\int_0^\tau k dt}{\tau}$$

$$\Rightarrow \bar{k} = 2\sqrt{\frac{D}{\pi \tau}} = 2k(\tau)$$



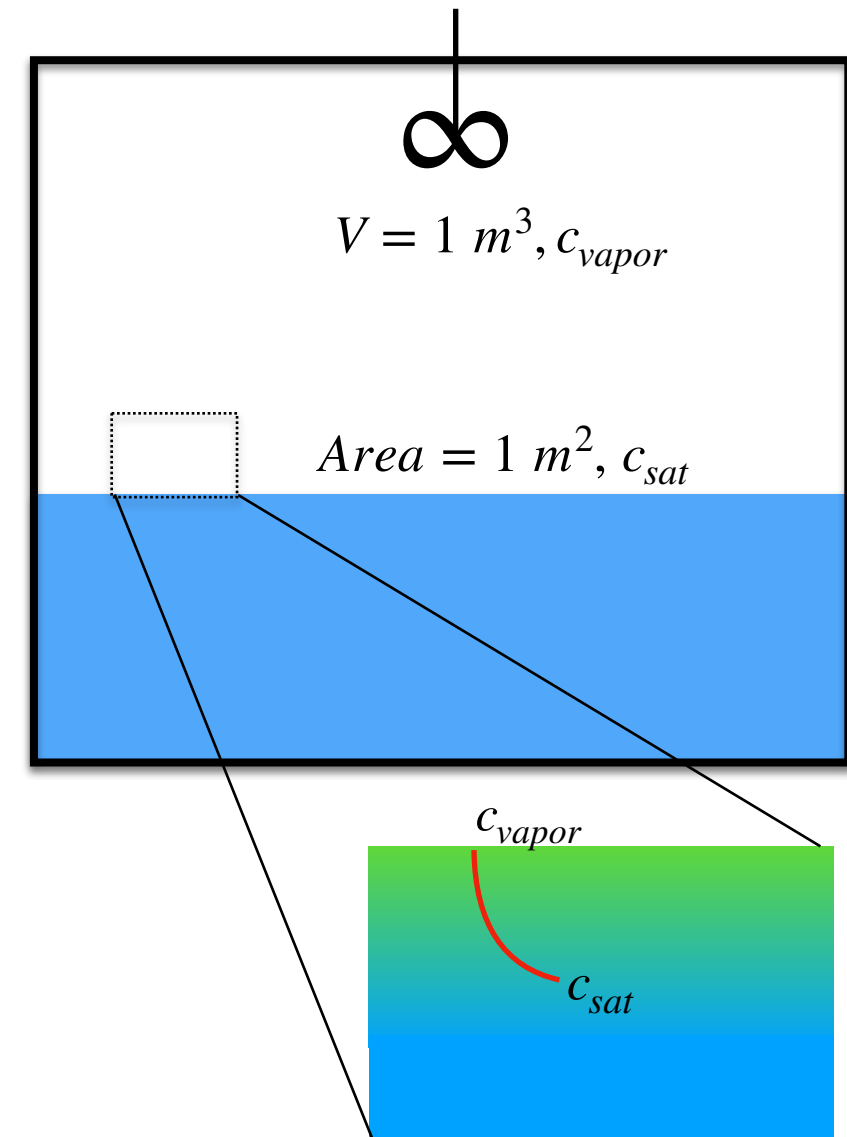
Humidification problem (liquid-vapor interface)

Water is evaporating from a bath into initially dry, well-mixed air at 1 bar. After 3 minutes, the water vapor in air is 0.003 bar. The system is isothermal at 25 °C. Saturation pressure for water is 0.03 bar.

Calculate k for transport of water vapor to air.
Also calculate time to reach 90% saturation.

$$N_1 = k(c_{1,i} - c_1)$$

Vapor accumulation in air = Mass transferred from liquid water



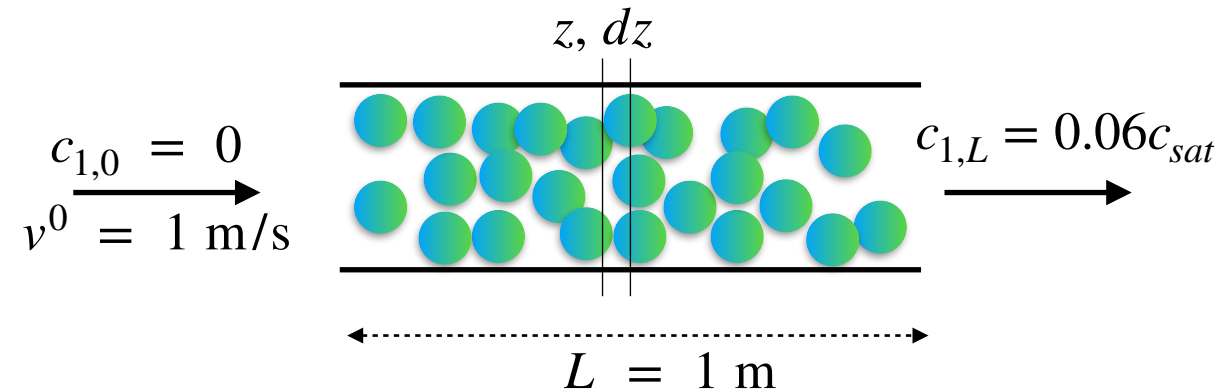
Packed bed problem (solid-liquid interface)

Pure water is flowing through a packed bed (1 m in length) made of benzoic acid beads to dissolve benzoic acid. The beads are 2 mm in diameter and have a mass transfer area of 20 cm² per cm³ volume of bed). If water comes out with a concentration of 6% saturation in steady-state conditions, calculate k for transport of benzoic acid to water.

a = surface area per unit volume for mass exchange

$$= \frac{20 \text{ cm}^2}{1 \text{ cm}^3} = 20 \text{ cm}^{-1}$$

How would you solve this problem?

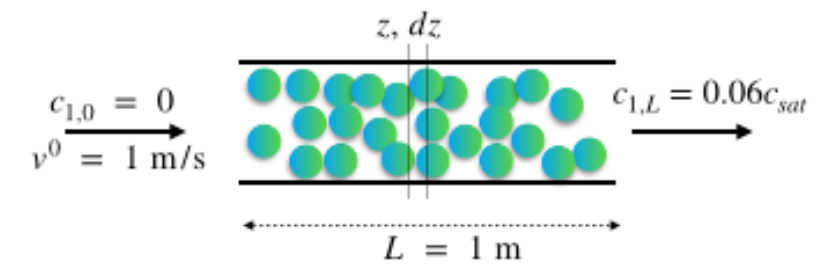


Active exchange area $A = aV$ where V is the volume of cylinder

We can do mass balance at length z in an element of width dz , $dV = Adz$

Packed bed problem

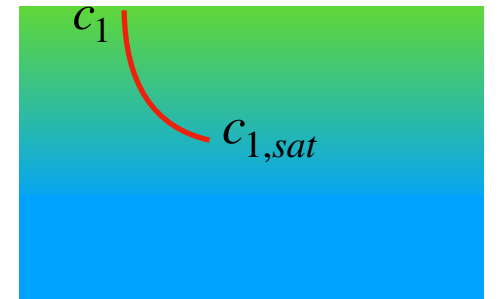
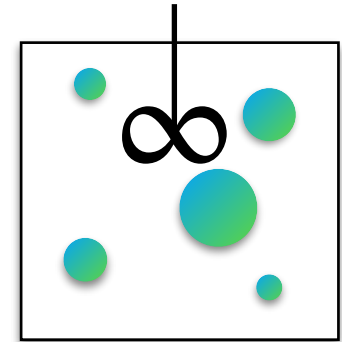
Accumulation in water = (flow in – flow out) + (mass transferred)



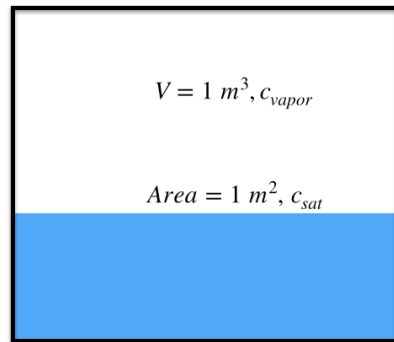
Emulsion problem (liquid-liquid interface)

Liquid bromine is rapidly dissolved in water by emulsifying liquid bromine droplets with water (mass exchange of bromine from droplet to bulk water). In 3 minutes, the concentration of bromine in water reaches 50% of saturation. Calculate ka where a is area of droplet per unit volume. Assume volume of water is constant, and a constant droplet size.

Accumulation in water = mass transferred

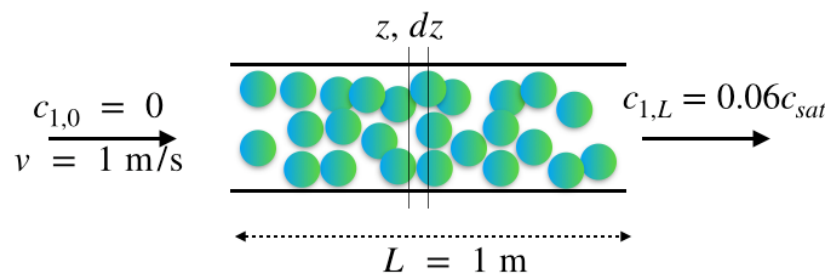


Analysis of previous 3 problems



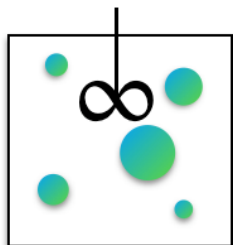
Liquid-vapor interface

$$c_{vapor} = c_{sat}(1 - e^{\frac{-kA}{V}t})$$



Solid-liquid interface

$$c_1 = c_{1,sat}(1 - e^{\frac{-ka}{v}z})$$



Liquid-liquid interface

$$c_1 = c_{sat}(1 - e^{-kat})$$

Dissolving gas bubbles (gas-liquid interface)

A 2 cm diameter oxygen bubble is injected in a well-mixed infinitely large water reservoir. If bubble shrinks by 50% of its diameter in 10 minutes, calculate k for transport of oxygen in water. Saturation concentration of dissolved oxygen is 0.0015 M.

How much will the bubble shrink in 20 minutes?

Analysis on the bubble

Mass loss of bubble = mass transferred

$$\frac{d}{dt}(c_1 V) = -k_g A(c_1 - c_{1,sat,g}) = -k_l A(c_{1,sat,l} - 0)$$

$c_{1,sat,g}$ is not known

c_1 is constant

$$\Rightarrow \frac{dV}{dt} = -k_l A \left(\frac{c_{1,sat,l}}{c_1} \right)$$

$$\Rightarrow \frac{d\left(\frac{4}{3}\pi r^3\right)}{dt} = -k_l (4\pi r^2) \left(\frac{c_{1,sat,l}}{c_1} \right)$$

$$\Rightarrow \frac{dr}{dt} = -k_l \frac{c_{1,sat,l}}{c_1}$$

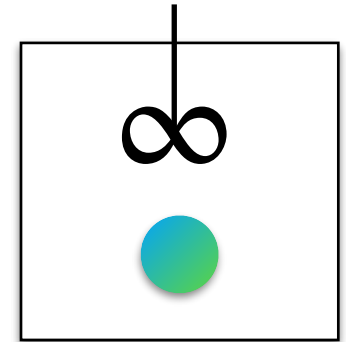
Integrate with initial condition, $t=0, r=r_0$

$$\Rightarrow r = r_0 - k_l \frac{c_{1,sat,l}}{c_1} t$$

$$c_1 = \frac{P}{RT} = \frac{10^5}{8.314 * 298} = 40.4 \text{ mole/m}^3$$

$$c_{1,sat,l} = 0.0015 \text{ M} = 1.5 \text{ mole/m}^3$$

$$k_l = 2.2 * 10^{-4} \text{ ms}^{-1}$$



Other representation of mass transfer coefficient

Representation

Units of mass transfer coefficient

$$N_1 = k \Delta c_1$$

$$\text{m s}^{-1}$$

$$N_1 = k_x \Delta x_1$$

$$\text{mol m}^{-2} \text{ s}^{-1}$$

$$N_1 = k_p \Delta P_1$$

$$\text{mol m}^{-2} \text{ Pa}^{-1} \text{ s}^{-1}$$

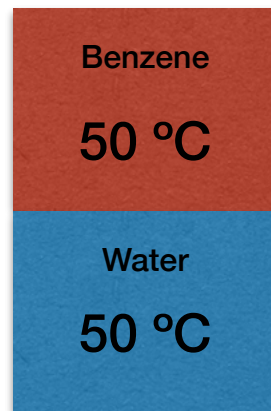
$$N_1 = k \Delta c_1 = k c \Delta x_1 = k_x \Delta x_1$$

$$N_1 = k \Delta c_1 = k \frac{1}{RT} \Delta P_1 = k_p \Delta P_1$$

$$\mathbf{k}_x = \mathbf{k} c \quad \text{m s}^{-1} \text{ mole m}^{-3}$$

$$\mathbf{k}_p = \frac{\mathbf{k}}{RT} \quad \frac{\text{ms}^{-1}}{\text{Pa m}^3 \text{ mole}^{-1}}$$

Is heat transfer possible



- A. Yes
- B. No

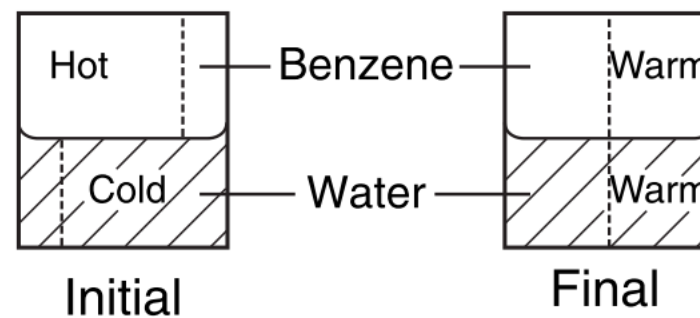
Is mass transfer possible (yes/no?)



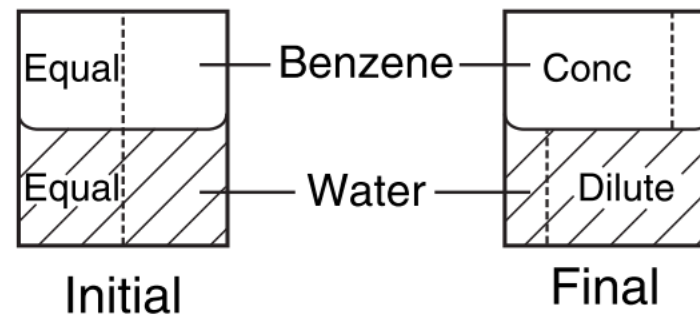
- A. Yes
- B. No

The driving force for heat transfer is temperature but for mass transfer is chemical potential.

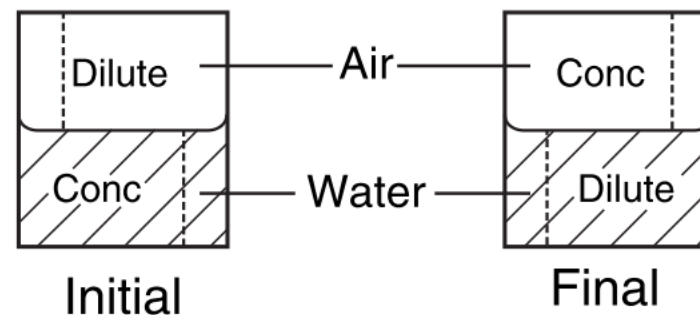
(a) Heat transfer



(b) Bromine extraction



(c) Bromine vaporization

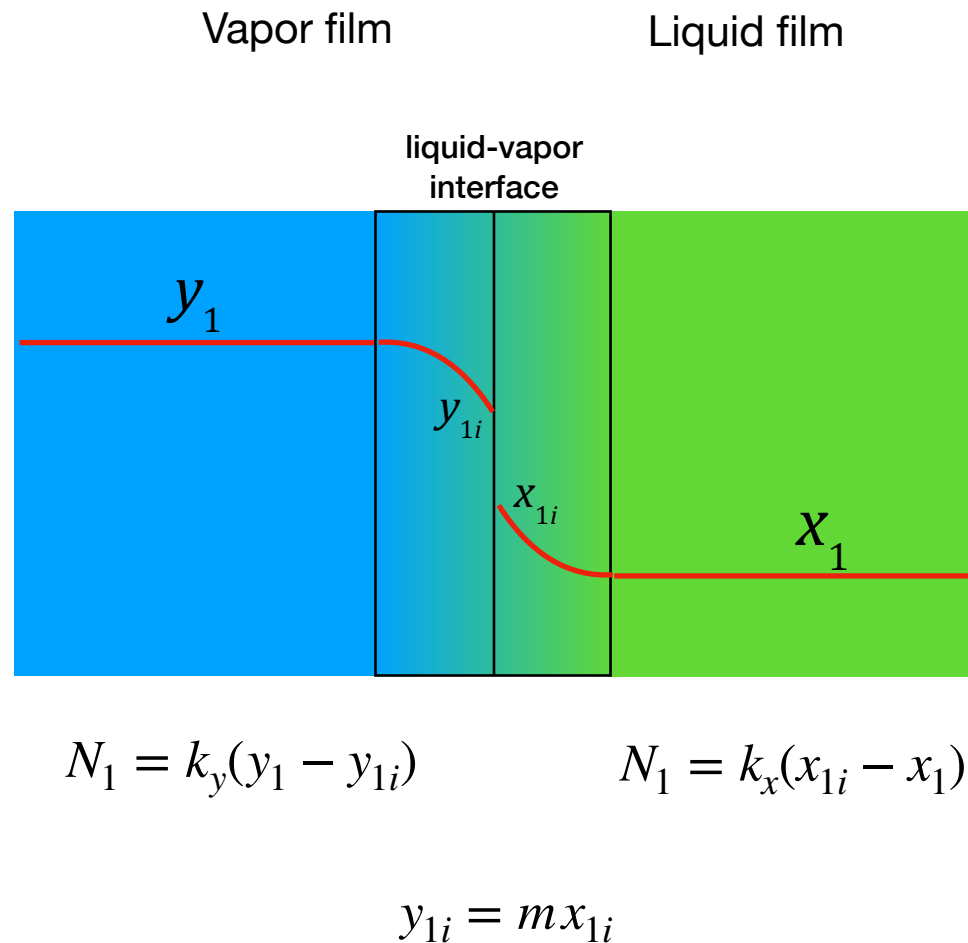


For an interface, how many mass transfer coefficient exist?

A. One

B. Two

Overall mass transfer coefficient between two phases in equilibrium



$$N_1 = k_y(y_1 - y_{1i})$$

$$N_1 = k_x(x_{1i} - x_1)$$

$$y_1 - y_{1i} = \frac{N_1}{k_y}$$

$$x_{1i} - x_1 = \frac{N_1}{k_x}$$

$$y_1 - mx_{1i} = \frac{N_1}{k_y}$$

$$mx_{1i} - mx_1 = \frac{mN_1}{k_x}$$

Add

$$y_1 - \cancel{mx_{1i}} + \cancel{mx_{1i}} - mx_1 = \frac{N_1}{k_y} + \frac{mN_1}{k_x}$$

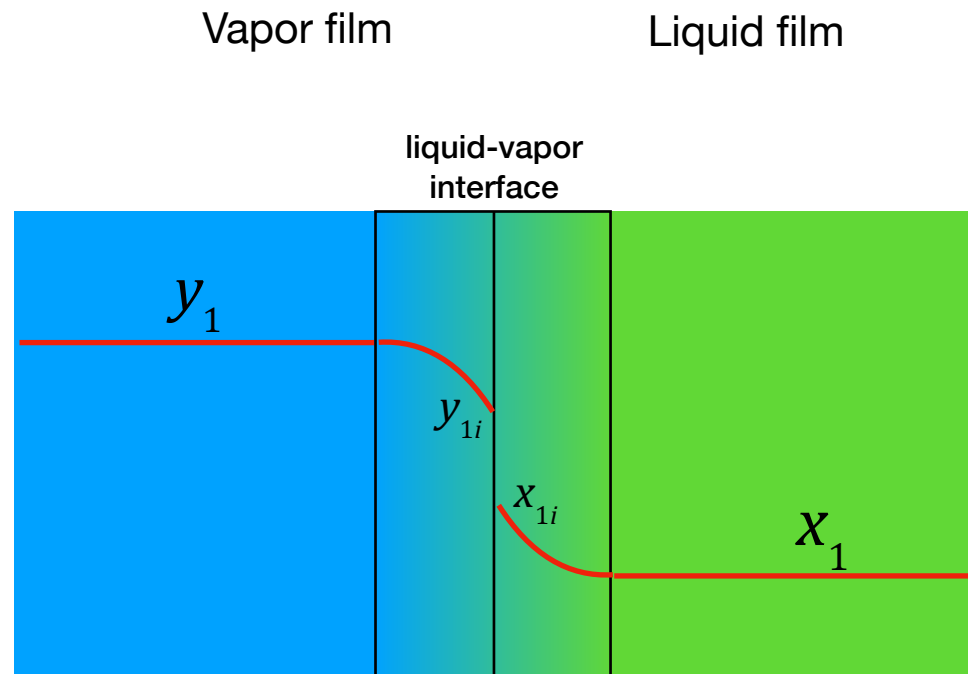
$$\Rightarrow y_1 - mx_1 = N_1 \left(\frac{1}{k_y} + \frac{m}{k_x} \right)$$

Usually y_{1i} and x_{1i} are not known

$$\Rightarrow y_1 - mx_1 = N_1 * \frac{1}{K_y} \quad \frac{1}{K_y} = \frac{1}{k_y} + \frac{m}{k_x}$$

$$\Rightarrow N_1 = K_y(y_1 - mx_1)$$

Overall mass transfer coefficient between two phases in equilibrium



$$N_1 = k_y(y_1 - y_{1i})$$

$$N_1 = k_x(x_{1i} - x_1)$$

$$y_{1i} = mx_{1i}$$

Usually y_{1i} and x_{1i} are not known

$$\frac{1}{K_y} = \frac{1}{k_y} + \frac{m}{k_x}$$

Prove

$$\frac{1}{K_x} = \frac{1}{mk_y} + \frac{1}{k_x}$$

What would be overall mass transfer coefficient from gas to liquid phase

Gas phase $N_1 = k_p(P_1 - P_{1i})$ $N_1 = K_p(P_1 - Hx_1) = K_x\left(\frac{P_1}{H} - x_1\right)$

Liquid phase $N_1 = k_x(x_{1i} - x_1)$

$$\frac{1}{K_p} = \frac{1}{k_p} + \frac{H}{k_x}$$

Equilibrium $H = \frac{P_{1i}}{x_{1i}}$

$$\frac{1}{K_x} = \frac{1}{Hk_p} + \frac{1}{k_x}$$

Few observations about overall coefficient

$$\frac{1}{K_y} = \frac{1}{k_y} + \frac{m}{k_x}$$

1. Total resistance = Phase I resistance + Phase II resistance
2. Higher resistance will dominate the overall transport.

$$k_y = 1$$

$$k_x = 10^{-4}$$

$$m = 1$$

$$K_y = 10^{-4}$$

Liquid phase (slow k) dominates

Other notations

Sometime, you often come across

$$N = K_p(P_1 - P_1^*)$$

Where P_1^* is hypothetical pressure in equilibrium with bulk liquid

$$N = K_x(x_1^* - x_1)$$

Where x_1^* is hypothetical mole fraction in equilibrium with bulk gas

$$P_1^* = Hx_1$$

$$x_1^* = \frac{P_1}{H}$$

Note, this is not equilibrium relationship for interfacial concentration
But merely a hypothetical concentration for the sake of convenience.

How would you calculate mass transfer coefficient?

Mass transfer coefficients, k , do not have molecular basis as that of diffusion coefficient, D .

Why we need correlation for mass transfer coefficient

- ✦ Mass transfer coefficients, k , do not have molecular basis as that of diffusion coefficient, D .
- ✦ For example k (oxygen dissolving in water) will differ based on stirring, whereas D will not change.

The relevance of mass transfer coefficient in the chemical industry (process scale-up)



Small-scale (10 L) bioreactor

Laboratory studies



Large-scale (10000 L) bioreactor

Manufacturing

Dimensionless analysis

There are several industrially important problems of interest:

- 1) Fluid flow through a packed bed of particles (ion-exchange, adsorption, absorption, catalysis...).
- 2) Gas bubbles rising in a liquid tank (humidification, absorption, oxygenation).
- 3) Falling films (humidification).

- ✦ Typically, several experiments are done in a small prototype (at small scale) to correlate parameters for dimensionless correlations or validate if existing.
- ✦ Dimensionless correlations are applied for scale-up.
- ✦ The error in such correlation can be as low as 1%, and as high as 30% depending on correlation accuracy and type of interface (fluid-fluid vs fluid-solid).
- ✦ This method has been heavily inspired by heat transport analysis which has been carried out for a longer period of history for fluid-solid interfaces.

$$\frac{kl}{D} = (\text{constant}) (\text{Re})^x (\text{Sc})^y$$

Variables important to dimensionless analysis

$D = \text{mass diffusivity} =$

$\alpha = \text{thermal diffusivity} = \frac{\text{heat conducted}}{\text{heat stored}}$

$\nu = \text{momentum diffusivity} =$

Schmidt number = $\frac{\nu}{D}$

$\frac{\text{momentum}}{\text{mass}}$

Lewis number = $\frac{\alpha}{D}$

$\frac{\text{heat}}{\text{mass}}$

Prandtl number = $\frac{\nu}{\alpha}$

$\frac{\text{momentum}}{\text{heat}}$

What other variables that come to your mind

k = mass transfer coefficient = velocity of diffusion =

v = velocity =

D = mass diffusivity =

Length-scale (thickness, diameter) matters

$$\text{Stanton number} = \frac{k}{v}$$

$$\frac{\text{velocity of diffusion}}{\text{velocity}}$$

$$\text{Sherwood number} = \frac{kl}{D}$$

$$\frac{\text{mass transfer velocity}}{\text{mass diffusivity}}$$

$$\text{Peclet number} = \frac{vl}{D}$$

$$\frac{\text{velocity}}{\text{mass diffusivity}}$$

Dimensional numbers important to mass transfer

Group ^a	Physical meaning	Used in
Sherwood number $\frac{kl}{D}$	$\frac{\text{mass transfer velocity}}{\text{diffusion velocity}}$	Usual dependent variable
Stanton number $\frac{k}{v^0}$	$\frac{\text{mass transfer velocity}}{\text{flow velocity}}$	Occasional dependent variable
Schmidt number $\frac{\nu}{D}$	$\frac{\text{diffusivity of momentum}}{\text{diffusivity of mass}}$	Correlations of gas or liquid data
Lewis number $\frac{\alpha}{D}$	$\frac{\text{diffusivity of energy}}{\text{diffusivity of mass}}$	Simultaneous heat and mass transfer

Dimensional numbers important to mass transfer

Group ^a	Physical meaning	Used in
Reynolds number $\frac{lv}{\nu}$	$\frac{\text{inertial forces}}{\text{viscous forces}}$ or $\frac{\text{flow velocity}}{\text{“momentum velocity”}}$	Forced convection
Grashof number $\frac{l^3 g \Delta \rho / \rho}{\nu^2}$	$\frac{\text{buoyancy forces}}{\text{viscous forces}}$	Free convection
Péclet number $\frac{v^0 l}{D}$	$\frac{\text{flow velocity}}{\text{diffusion velocity}}$	Correlations of gas or liquid data
Second Damköhler number or (Thiele modulus) ² $\frac{\kappa l^2}{D}$	$\frac{\text{reaction velocity}}{\text{diffusion velocity}}$	Correlations involving reactions (see Chapters 16–17)

Frequently used mass transfer correlations

Divided into 2 categories

Fluid-fluid interface

- ✖ Distillation
- ✖ Absorption
- ✖ Extraction
- ✖ Water aeration
- ✖ Oxygenation

Frequently used in chemical industry.
No parallel in heat transfer.

Error of the order of 30%

Fluid-solid interface

- ✖ Membranes
- ✖ Adsorption
- ✖ Leaching
- ✖ Catalyst bed

Heavily inspired from heat transfer.

Error at around 10%,
At best 1 % (laminar flow in a single tube)

Laminar flow of one fluid in a tube is much better understood than turbulent flow of gas and liquid in a packed tower

- ✖ Due to the errors, these correlations are often used to design a small-scale (pilot plant).
- ✖ Typically you will validate the result in pilot plant with the actual chemicals, and then make a scale-up model.

Fluid-fluid interface are difficult to handle

- ✦ Re can vary by a factor of 10000 (laminar vs. turbulent flow). $Re = \frac{dv\rho}{\eta}$
- ✦ Correlation involve velocities. Velocities have 2 origins.
 - Forced (stirring, pumping).
 - Free (density gradients, gravity).
- ✦ Sc in gases ~ 1 , in liquids ~ 1000 .

Water at 25 °C as an example:

Liquid:	$\eta = 0.001 \text{ Pa s}$	$\rho = 1000 \text{ Kg/m}^3$	$D = 10^{-9} \text{ m}^2/\text{s}$	$Sc =$
Vapor:	$\eta = 10^{-5} \text{ Pa s}$	$\rho = 1 \text{ Kg/m}^3$	$D = 10^{-5} \text{ m}^2/\text{s}$	$Sc =$

How dimensionless correlations are made

$$ka = ka(v, \rho, \mu, d, z)$$

Assume the function has an exponential form

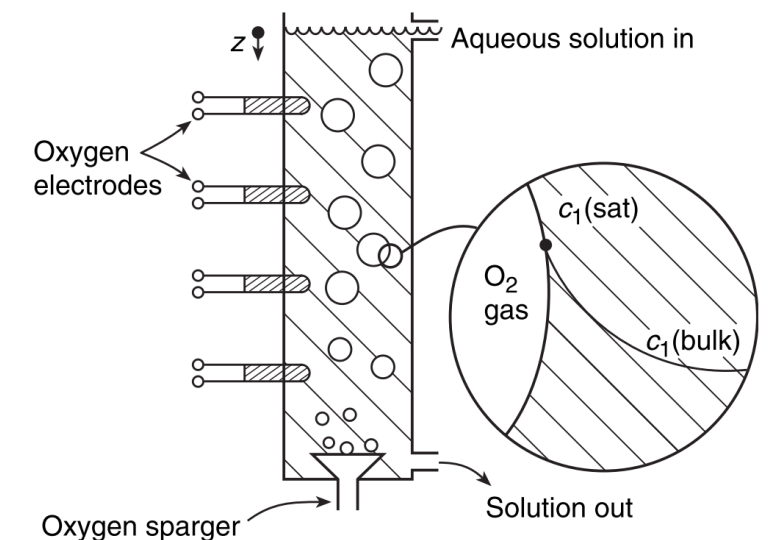
$$ka = [\text{constant}] v^\alpha \rho^\beta \mu^\gamma d^\delta z^\epsilon$$

Check dimension on both sides

$$\frac{L}{t} \frac{1}{L} [=] \left(\frac{L}{t} \right)^\alpha \left(\frac{M}{L^3} \right)^\beta \left(\frac{M}{Lt} \right)^\gamma L^\delta L^\epsilon$$

$$t^{-1} [=] L^{(\alpha-3\beta-\gamma+\delta+\epsilon)} t^{(-\alpha-\gamma)} M^{(\beta+\gamma)}$$

The only way this equation can be dimensionally consistent is if the exponent on time on the left-hand side of the equation equals the sum of the exponents on time on the right-hand side



How dimensionless correlations are made

$$t^{-1} [=] L^{(\alpha-3\beta-\gamma+\delta+\epsilon)} t^{(-\alpha-\gamma)} M^{(\beta+\gamma)}$$

$$\Rightarrow \alpha - 3\beta - \gamma + \delta + \epsilon = 0$$

$$\Rightarrow -\alpha - \gamma = -1$$

$$\Rightarrow \beta + \gamma = 0$$

3 equations, 5 variables

At least 2 dependent variables

If we arbitrarily choose γ and ϵ as the dependent variable

$$\alpha = 1 - \gamma$$

$$\beta = -\gamma$$

$$\gamma = \gamma$$

$$\delta = -\gamma - \epsilon - 1$$

$$\epsilon = \epsilon$$

$$ka = [\text{constant}] v^\alpha \rho^\beta \mu^\gamma d^\delta z^\epsilon$$

$$\Rightarrow ka = [\text{constant}] v^{(1-\gamma)} \rho^{(-\gamma)} \mu^\gamma d^{(-\gamma-\epsilon-1)} z^\epsilon$$

$$\Rightarrow ka = [\text{constant}] \frac{v}{v^\gamma} \frac{1}{\rho^\gamma} \mu^\gamma \frac{1}{d^\gamma} \frac{1}{d^\epsilon} z^\epsilon$$

How dimensionless correlations are made

$$\Rightarrow ka = [\text{constant}] \frac{v}{v^\gamma} \frac{1}{\rho^\gamma} \mu^\gamma \frac{1}{d^\gamma} \frac{1}{d^\epsilon} z^\epsilon$$

$$\Rightarrow \frac{kad}{v} = [\text{constant}] \left(\frac{\mu}{dv\rho} \right)^\gamma \left(\frac{z}{d} \right)^\epsilon$$

$$St = [\text{constant}] Re^{-\gamma} \left(\frac{z}{d} \right)^\epsilon$$

$$\text{Stanton number} = \frac{k}{v}$$

$$\text{Reynolds number} = \frac{dv\rho}{\mu}$$

Experimental method

Plot Stanton number vs Re and z/d to get the exponents

Challenges in such analysis

- ✘ Will only work if we select the right key variable (in this case, γ and ϵ)
- ✘ Assumption that the bulk liquid is well mixed
- ✘ Data may not fit the exponential form
- ✘ We usually do not know at the start as to which variables are important (too many, too less)

Frequently used mass transfer correlations

General patterns: correlation of *Sh (or St)* to *Re and Sc*

$$\text{Sherwood number} = \frac{kl}{D}$$

$$\text{Reynolds number} = \frac{dv}{\nu}$$

$$\text{Stanton number} = \frac{k}{v}$$

$$\text{Schmidt number} = \frac{\nu}{D}$$

Packed beds

$$\frac{k}{v^0} = 1.17 \left(\frac{dv^0}{\nu} \right)^{-0.42} \left(\frac{D}{\nu} \right)^{2/3}$$

d = particle diameter
 v^0 = superficial velocity

The superficial velocity is that which would exist without packing

Can you write above correlation in terms of *Sh (or St)* as a function of *Sc*, and *Re* ??

$$St = 1.17 Re^{-0.42} Sc^{-2/3}$$

Fluid-fluid interface (Packed Tower)

Physical situation	Basic equation ^b	Key variables	Remarks
Liquid in a packed tower	$k \left(\frac{1}{\nu g} \right)^{1/3} = 0.0051 \left(\frac{v^0}{a\nu} \right)^{0.67} \left(\frac{D}{\nu} \right)^{0.50} (ad)^{0.4}$	a = packing area per bed volume d = nominal packing size	Probably the best available correlation for liquids; tends to give lower value than other correlations
	$\frac{kd}{D} = 25 \left(\frac{dv^0}{\nu} \right)^{0.45} \left(\frac{\nu}{D} \right)^{0.5}$	d = nominal packing size	The classical result, widely quoted; probably less successful than above
Gas in a packed tower	$\frac{k}{aD} = 3.6 \left(\frac{v^0}{a\nu} \right)^{0.70} \left(\frac{\nu}{D} \right)^{1/3} (ad)^{-2.0}$	a = packing area per bed volume d = nominal packing size	Probably the best available correlation for gases
	$\frac{kd}{D} = 1.2 (1 - \varepsilon)^{0.36} \left(\frac{dv^0}{\nu} \right)^{0.64} \left(\frac{\nu}{D} \right)^{1/3}$	d = nominal packing size ε = bed void fraction	Again, the most widely quoted classical result

$k \left(\frac{1}{\nu g} \right)^{1/3}$ is unusual form of St

v^0 is superficial velocity (Velocity that exists without packing)

Can you write above correlations in terms of Sh (or St) and Re and Sc ?

Fluid-fluid interface (Gas bubbles in tank)

Physical situation	Basic equation ^b	Key variables	Remarks
Pure gas bubbles in a stirred tank	$\frac{kd}{D} = 0.13 \left(\frac{(P/V) d^4}{\rho \nu^3} \right)^{1/4} \left(\frac{\nu}{D} \right)^{1/3}$	d = bubble diameter P/V = stirrer power per volume	Note that k does not depend on bubble size
Pure gas bubbles in an unstirred tank	$\frac{kd}{D} = 0.31 \left(\frac{d^3 g \Delta \rho / \rho}{\nu^2} \right)^{1/3} \left(\frac{\nu}{D} \right)^{1/3}$	d = bubble diameter $\Delta \rho$ = density difference between bubble and surrounding fluid	0.3-cm diameter or larger

$\frac{d^3 g (\Delta \rho / \rho)}{\nu^2}$ is Grashof number

Fluid-fluid interface (some others)

Physical situation	Basic equation ^b	Key variables	Remarks
Small liquid drops rising in unstirred solution	$\frac{kd}{D} = 1.13 \left(\frac{dv^0}{D} \right)^{0.8}$	d = drop diameter v^0 = drop velocity	These small drops behave like rigid spheres
Falling films	$\frac{kz}{D} = 0.69 \left(\frac{zv^0}{D} \right)^{0.5}$	z = position along film v^0 = average film velocity	Frequently embroidered and embellished

What dimensionless number are these ?

$$\text{Peclet number} = \frac{vl}{D}$$

$$\frac{\text{velocity}}{\text{mass diffusivity}}$$

Fluid-solid interface

Table 8.3-3 Selected mass transfer correlations for fluid-solid interfaces^a

Physical situation	Basic equation ^b	Key variables	Remarks
Membrane	$\frac{kl}{D} = 1$	l = membrane thickness	Often applied even where membrane is hypothetical
Laminar flow along flat plate ^c	$\frac{kL}{D} = 0.646 \left(\frac{Lv^0}{\nu} \right)^{1/3} \left(\frac{\nu}{D} \right)^{1/3}$	L = plate length v^0 = bulk velocity	Solid theoretical foundation, which is <i>Re</i> < 2000 unusual
Turbulent flow through horizontal slit	$\frac{kd}{D} = 0.026 \left(\frac{dv^0}{\nu} \right)^{0.8} \left(\frac{\nu}{D} \right)^{1/3}$	v^0 = average velocity in slit $d = [2/\pi]$ (slit width)	Mass transfer here is identical with that in a pipe of equal wetted perimeter
Turbulent flow through circular pipe	$\frac{kd}{D} = 0.026 \left(\frac{dv^0}{\nu} \right)^{0.8} \left(\frac{\nu}{D} \right)^{1/3}$	v^0 = average velocity in slit d = pipe diameter	Same as slit, because only wall regime is involved
Laminar flow through circular tube	$\frac{kd}{D} = 1.62 \left(\frac{d^2 v^0}{LD} \right)^{1/3}$	d = pipe diameter L = pipe length v^0 = average velocity in tube	Very strong theoretical and experimental basis <i>Re</i> < 2000
Flow outside and parallel to a capillary bed	$\frac{kd}{D} = 1.25 \left(\frac{d^2 v^0}{\nu l} \right)^{0.93} \left(\frac{\nu}{D} \right)^{1/3}$	d = 4 cross-sectional area/(wetted perimeter) v^0 = superficial velocity	Not reliable because of channeling in bed
Flow outside and perpendicular to a capillary bed	$\frac{kd}{D} = 0.80 \left(\frac{dv^0}{\nu} \right)^{0.47} \left(\frac{\nu}{D} \right)^{1/3}$	d = capillary diameter v^0 = velocity approaching bed	Reliable if capillaries evenly spaced
Forced convection around a solid sphere	$\frac{kd}{D} = 2.0 + 0.6 \left(\frac{dv^0}{\nu} \right)^{1/2} \left(\frac{\nu}{D} \right)^{1/3}$	d = sphere diameter v^0 = velocity of sphere	Very difficult to reach $(kd/D) = 2$ experimentally; no sudden laminar-turbulent transition
Free convection around a solid sphere	$\frac{kd}{D} = 2.0 + 0.6 \left(\frac{d^3 \Delta \rho g}{\rho \nu^2} \right)^{1/4} \left(\frac{\nu}{D} \right)^{1/3}$	d = sphere diameter g = gravitational acceleration	For a 1-cm sphere in water, free convection is important when $\Delta \rho = 10^{-9}$ g/cm ³
Packed beds	$\frac{k}{v^0} = 1.17 \left(\frac{dv^0}{\nu} \right)^{-0.42} \left(\frac{D}{\nu} \right)^{2/3}$	d = particle diameter v^0 = superficial velocity	The superficial velocity is that which would exist without packing
Spinning disc	$\frac{kd}{D} = 0.62 \left(\frac{d^2 \omega}{\nu} \right)^{1/2} \left(\frac{\nu}{D} \right)^{1/3}$	d = disc diameter ω = disc rotation (radians/time)	Valid for Reynolds numbers between 100 and 20,000

Notes: ^a The symbols used include the following: D is the diffusion coefficient of the material being transferred; k is the local mass transfer coefficient; ρ is the fluid density; ν is the kinematic viscosity. Other symbols are defined for the specific situation.

^b The dimensionless groups are defined as follows: (dv^0/ν) and $(d^2\omega/\nu)$ are the Reynolds number; ν/D is the Schmidt number; $(d^3\Delta\rho g/\rho\nu^2)$ is the Grashof number, kd/D is the Sherwood number; k/v^0 is the Stanton number.

^c The mass transfer coefficient given here is the value averaged over the length L .